

String Theories with Moduli Stabilization Imply Non-Thermal Cosmological History, and Particular Dark Matter.

Bobby Samir Acharya^{b,a}, Gordon Kane^a, and Eric Kuflik^a

^aMichigan Center for Theoretical Physics, University of Michigan, Ann Arbor, MI 48109

^bAbdus Salam International Center for Theoretical Physics, Trieste, Italy

(Dated: October 13, 2010)

In recent years it has been realised that pre-BBN decays of moduli can be a significant source of dark matter production, giving a ‘non-thermal WIMP miracle’ and substantially reduced fine-tuning in cosmological axion physics. We study moduli masses and sharpen the claim that moduli dominated the pre-BBN Universe. We conjecture that in any string theory with stabilized moduli there will be at least one modulus field whose mass is of order (or less than) the gravitino mass. Cosmology then generically requires the gravitino mass not be less than about 30 TeV and the cosmological history of the Universe is non-thermal prior to BBN. Stable LSP’s produced in these decays can account for the observed dark matter if they are ‘wino-like,’ which is consistent with the PAMELA data for positrons and antiprotons. With WIMP dark matter, there is an upper limit on the gravitino mass of order 250 TeV. We briefly consider implications for the LHC, rare decays, and dark matter direct detection and point out that these results could prove challenging for models attempting to realize gauge mediation in string theory.

INTRODUCTION

String theories have moduli – scalar fields with no classical potential and with Planck scale suppressed couplings to matter. The expectation values of the moduli classically describe the size and configuration of the curled up extra dimensions. In order to have a meaningful model describing phenomena below the string scale, the moduli must be “stabilized”, i.e., must have a potential with a minimum that determines their value in the vacuum. Otherwise observable coupling strengths and masses would not have meaningful values when calculated from the theory, making it difficult to compare with data.

In a vacuum in which local supersymmetry is spontaneously broken there is a goldstone fermion (goldstino) that becomes the longitudinal spin state of the spin 3/2 gravitino, so that the gravitino is massive. The gravitino mass - $m_{3/2}$ - typically sets the scale for all of the scalar masses in the theory. Therefore, one generically expects moduli masses of order $m_{3/2}$, far below the string scale. One of the aims of this paper is to sharpen this last statement.

One expects that after inflation the Hubble scale $H_I \geq m_{3/2}$, and we will assume that this is true throughout this paper. Eventually, H will decrease until $H \sim m_\phi$, the modulus mass, at which point the moduli will begin to oscillate in their potential. One can estimate the energy stored in the oscillating moduli, $\rho_\phi \sim m_\phi^2 m_{pl}^2$ which has long been known [1] to be large and to dominate the energy density of the Universe. Since the moduli quanta are particles the Universe is matter dominated during these oscillations, not radiation dominated. The moduli (ϕ) couple gravitationally to all other matter, with decay widths $\Gamma_\phi \sim d_\phi m_\phi^3 / m_{pl}^2$, where d_ϕ is a number of order unity that is in principle calculable in any particular theory (see [2, 3] for examples). Therefore, their lifetimes and decays can be estimated.

If the moduli decay after about 10^{-2} seconds after inflation ends, their decay products will inject additional photons, hadrons and leptons during Big Bang Nucleosynthesis

(BBN), ruining its successful predictions. Lifetimes shorter than about 10^{-2} seconds require moduli masses larger than about 30 TeV [4]. If they do not decay until after BBN we can estimate the energy stored in them from the energy density-to-entropy ratio ρ/s , which is approximately constant as the Universe cools. Today, ρ/s is of order a few eV, but for the moduli it is of order T_{osc} , the temperature of the Universe at the onset of the oscillations, which is as large as 10^{11} GeV for moduli masses of order TeV. Since this is many orders of magnitude larger than ρ/s today, the moduli must decay before BBN. This is the cosmological moduli problem [1]. The only known way to avoid this result is to have a (tuned) period of late inflation, called thermal inflation [5–7] which could dilute the energy density of the moduli fields. An interesting question is whether or not thermal inflation might occur “naturally” in string theory, but we will leave this for future work. In the following we will assume thermal inflation does not occur.

In recent years, however, it has been realised that models with moduli which decay before BBN can have virtues which are comparable to, or improvements upon, models which have a ‘thermal cosmological history’. There can be a ‘non-thermal WIMP miracle’ which is equally compelling as the thermal case [2, 8] and requires larger WIMP annihilation cross-sections (which happen to be better suited for explaining the PAMELA data [9, 10]). Further, the entropy released from the moduli decays dilutes potential axion relic abundances and allows for much less fine-tuned cosmological axion physics than is the case in a ‘thermal cosmological history’, thereby relieving the tension between cosmological bounds and GUT scale axion decay constants [11]. These virtues of a ‘non-thermal cosmological history’ indicate that the Cosmological Moduli Problem is, perhaps, less of a problem and more likely part of a solution. Given the potential impact of such an indication, it is of importance to investigate in more detail the claim that *generically the moduli masses will be of order $m_{3/2}$* .

We will try to sharpen some of the existing arguments that realistic vacua arising from a compactified string theory will, generically, have moduli or moduli-like fields (such as hidden sector matter scalars, or axions), which dominate the energy density of the Universe prior to BBN. With moduli stabilized, the moduli F -terms and those of the moduli-like fields contribute to supersymmetry breaking, so the scalar goldstino will have significant moduli components. We will argue that one or more moduli or moduli-like fields will have masses of order the gravitino mass. These results can apply regardless of the value of the gravitino mass and, hence, can give a strong constraint on the model of mediation of supersymmetry breaking. We also discuss the moduli spectrum in a variety of different classes of vacua in which moduli stabilization is fairly well understood and demonstrate that there is always a modulus-like field which dominates the cosmic energy density prior to BBN. These latter results are described in Appendix B.

We have thus given further evidence that, from the string theory point of view, the Universe is expected to have a partly non-thermal history – an important claim for considerations of cosmology, especially dark matter. The gravitino mass is required to be greater than about 30 TeV implying that gauge mediated supersymmetry breaking will be difficult to realise in a phenomenologically consistent string vacuum, unless one can dilute the moduli energy density through late inflation [6].

The next section describes the main results which are based on general considerations of supergravity and follow the earlier approach of [12]. The following section describes the main physics implications of the result.

SUPERGRAVITY AND MODULI MASSES

The conjecture: In any string/ M theory vacuum with observationally consistent energy density of the Universe, there exists at least one modulus-like field whose mass is such that it dominates the vacuum energy up to the BBN era.

Here, the term modulus-like refers not only to the geometric moduli fields of string theory, but includes other scalar fields whose couplings to Standard Model particles are suppressed by a high scale such as the Grand Unification scale or Planck scale. Examples include axions and other hidden sector fields.

Typically, as we will see, the conjecture follows from the fact that the masses of these moduli-like fields are of order, or less than, the gravitino mass $m_{3/2}$; in fact this would be a more general version of the conjecture. As we will see below, there can be examples in which the relevant mass scale is much less than $m_{3/2}$ because of “large volume” effects in the extra dimensional theory [14]. These examples do not violate either conjecture.

Since the moduli fields are stabilized by assumption, they have non-trivial potentials and will most likely participate in supersymmetry breaking. If this is true, i.e., at least one moduli has a non-vanishing F -term, then one can make progress towards proving the conjecture by considering the scalar potential of the effective supergravity theory. (String vacua in

which the 4-D supergravity approximation is not valid may be beyond the scope of this paper).

The scalar potential in a supergravity theory in four dimensions can be written in terms of a single real function G . In terms of the Kahler potential and superpotential $G = K + m_{pl}^2 \ln(W\bar{W}/m_{pl}^6)$, though we will only consider G here. G is taken to have mass dimension two and all scalar fields are taken to be dimensionless in order to exhibit correctly the scaling of various operators with the reduced Planck mass m_{pl} . The scalar field potential, assuming that the D -term contributions are negligible is

$$V = m_{pl}^2 e^{G/m_{pl}^2} (G^i G_i - 3m_{pl}^2) \quad (1)$$

Critical points of V satisfy

$$\langle \nabla_i V \rangle = \left\langle m_{pl}^{-2} G_i V + e^{G/m_{pl}^2} (G_i + G^k \nabla_i G_k) \right\rangle = 0. \quad (2)$$

where subscripts on G denote derivatives with respect to the moduli fields ϕ_i or their conjugates ϕ_i^* .

One can calculate the complex scalar mass matrix in the vacua which satisfy the above conditions and, additionally, have zero cosmological constant. Assuming the kinetic terms have been properly normalized, the mass matrix for all the scalar fields in the theory is

$$\begin{aligned} M_{ij}^2 &= e^{G/m_{pl}^2} \left(\nabla_i G_k \nabla_{\bar{j}} G^k - R_{i\bar{j}k\bar{l}} G^k G^{\bar{l}} + G_{i\bar{j}} \right) \\ M_{i\bar{j}}^2 &= e^{G/m_{pl}^2} (2\nabla_i G_{\bar{j}} + G^k \nabla_i \nabla_{\bar{j}} G_k) \end{aligned} \quad (3)$$

Notice that there is no factor of m_{pl}^2 because the quantities in the brackets all have mass dimension two.

Since we are only interested in minima of the potential, the mass matrix is positive definite by assumption. Hence, we use the theorem (see Appendix C) that its smallest eigenvalue, m_{\min}^2 , is less than $\xi^\dagger M \xi$ for any unit vector ξ . Extending the work of [12], we take $\xi = (G^{\bar{j}} \quad c G^j) / \sqrt{3(1+|c|^2)}$ for $c \in \mathbb{C}$, which is aligned in the (moduli components of the) two sGoldstino directions $\eta = G^i \phi_i$ and $\bar{\eta} = G_{\bar{i}} \phi^{\bar{i}}$. This gives a one (complex) parameter class of constraints on the upper bound of the lowest mass eigenvalue

$$\begin{aligned} m_{\min}^2 &\leq \frac{1}{3(1+|c|^2)} \begin{pmatrix} G^i & c^\dagger G^{\bar{i}} \end{pmatrix} \begin{pmatrix} M_{ij}^2 & M_{i\bar{j}}^2 \\ M_{\bar{i}j}^2 & M_{\bar{i}\bar{j}}^2 \end{pmatrix} \begin{pmatrix} G^{\bar{j}} \\ c G^j \end{pmatrix} \\ &\leq m_{3/2}^2 \left(2 \frac{|1-c|^2}{1+|c|^2} + \text{Re} \left\{ \frac{2c}{1+|c|^2} \frac{u}{m_{pl}^2} \right\} - \frac{r}{m_{pl}^2} \right) \end{aligned} \quad (4)$$

where $u \equiv \frac{1}{3} G^i G^{\bar{j}} G^k \nabla_i \nabla_{\bar{j}} G_k$, $r \equiv \frac{1}{3} R_{i\bar{j}k\bar{l}} G^i G^{\bar{j}} G^k G^{\bar{l}}$ and $m_{3/2}^2 = m_{pl}^2 e^{G/m_{pl}^2}$. r is the holomorphic sectional curvature of the scalar field space, evaluated in the sGoldstino directions in field space. We have extended the previous work to include the effects of the curvature r as well as u .

To understand the constraint given by Eq. (4) we rewrite

this equation by taking $u = |u|e^{i\theta_u}$

$$m_{\min}^2 < m_{3/2}^2 \left(2 - 2\alpha \cos \theta + \alpha \frac{|u|}{m_{pl}^2} \cos(\theta + \theta_u) - \frac{r}{m_{pl}^2} \right) \quad (5)$$

for any $\alpha \equiv \frac{2|c|}{1+|c|^2} \in [0, 1]$ and $\theta \in [0, 2\pi]$.

It therefore follows that

$$m_{\min}^2 < m_{3/2}^2 \left(2 - \frac{r}{m_{pl}^2} \right) \quad (6)$$

and for $u \in \mathbb{R}$

$$m_{\min}^2 < m_{3/2}^2 \left(\min\left\{ \frac{u}{m_{pl}^2}, 4 - \frac{u}{m_{pl}^2} \right\} - \frac{r}{m_{pl}^2} \right). \quad (7)$$

So, as long as $|r|, |u| \leq O(m_{pl}^2)$, the upper limit on the lightest modulus mass is of order the gravitino mass $m_{3/2}$. (See Appendix A for a simple model illustrating the bound given by Eq. (6) and Eq. (7)).

In fact, for geometric moduli, r is typically of order m_{pl}^2 , and thus there will generically be at least one moduli with mass $\lesssim m_{3/2}$. In Appendix B we systematically discuss the moduli masses in all known (at least to us) examples where moduli stabilization is well understood. It is demonstrated that all of these examples have $r \sim m_{pl}^2$ and a modulus or modulus-like field whose mass is less than, or of order $m_{3/2}$

NON-GENERIC POSSIBILITIES

One can discuss under what non-generic conditions moduli, or moduli-like fields, will *not* dominate the cosmic energy density prior to BBN. One possibility is that *all* moduli that have mass order $m_{3/2}$ have significant mixing with charged (under the SM gauge groups or other gauge groups) scalar fields. Then the lightest eigenvalue given by Eq. (7) and Eq. (6) can have significant charged matter components and quickly thermalises due to its couplings to gauge fields and matter. However, mixing between moduli and matter is proportional to vevs of the matter fields or their F -terms and are usually suppressed. It would be difficult to arrange for all moduli to have such large couplings to matter fields. But if any stabilized modulus field (or linear combination) does not mix, our results will hold. Another possibility is to have moduli stabilization unrelated to supersymmetry breaking and a stabilization mechanism that gives all moduli very large masses, but as stated earlier, non-trivial potentials that stabilize moduli will generally break supersymmetry. A third is to have r or u extremely large (and negative), e.g. $|r| \gg (30 \text{ TeV}/m_{3/2})^2 m_{pl}^2$, or have very large kinetic terms so that the mass matrix (Eq. 3) receives large scaling factors when the kinetic terms are properly normalized. At present such non-generic cases are not excluded, but any proposed model has to explain why they might occur. A final possibility is to have moduli of order $m_{3/2}$, but not oscillate in the

early universe, for example, see "moduli trapping" [18, 19] or "string gas cosmology" [20].

Part of the difficulty in trying to make such a model is that there is always at least one axion field present in four dimensional string theory vacua; usually, there are many axions [21]. Some of these axions will be much lighter than the moduli, which are assumed to be heavy enough to be cosmologically irrelevant in this part of the discussion. This is because axions only gain mass via non-perturbative effects. One can calculate the relic abundance of such an axion today, as a function of its mass, m_a , decay constant, f_a and initial displacement θ_a It is given by [11]

$$\Omega_a h^2 = 0.06 \left(\frac{f_a^2}{M_{GUT}^2} \right) \left(\frac{m_a}{10^{-20} \text{ eV}} \right)^{1/2} \langle \theta_a^2 \rangle \quad (8)$$

With a GUT scale decay constant, the axion lifetime extends into the BBN era if $m_a \leq 100 \text{ GeV}$, so the above formula implies that one would have to tune the model such that there are no axions with masses between 10^{-20} eV and 100 GeV !

NON-THERMAL COSMOLOGICAL HISTORY FROM STRING THEORY

The arguments based on the moduli mass matrix imply that $m_\phi \leq O(m_{3/2})$, and that $m_{3/2}$ must be of order 30 TeV or larger to not conflict with BBN predictions or the observed late energy density [22]. The moduli couple essentially universally to every Standard Model particle and their superpartners. Some moduli decays might be helicity suppressed, but decays to scalars are all present at full strength. These decays generate huge entropy, which significantly dilutes any dark matter particles that might have been present before the moduli decay. It is sufficient if only one such modulus has mass of order $m_{3/2}$ though typically many do. *Thus thermal freezeout relic densities of dark matter are not relevant to present cosmology in string models where moduli are present.* At the same time, about a quarter [2, 3] of all moduli decays will be to superpartners, and every superpartner will have a decay chain with a lightest superpartner (LSP) at the end, so a large number of LSP dark matter particles will be generated and provide a dark matter candidate if they are stable. In practice, the number density of LSP's from moduli decay is large compared to the relic density, large enough for LSP's to annihilate. The typical temperature after the moduli decay is of order 10 MeV , but the number density decreases as the Universe expands, approaching an attractor solution of the Boltzmann equations (when the number density is too small for annihilation to take place) and not a freeze-out (which occurs when the LSP's fall out of equilibrium). Surprisingly, one still finds a "WIMP miracle", where the relic number density is still given in terms of the Hubble parameter - a cosmological parameter, but now evaluated at the moduli decay temperature rather than a freezeout temperature - and a particle physics annihilation

cross-section for the LSP's (appropriately averaged) [8]

$$N_{LSP} \approx H(T \sim 10 \text{ MeV}) / \langle \sigma_{annih}^{LSP} v \rangle. \quad (9)$$

All steps of this calculation have been carried out in the example of M theory compactified on a manifold of G_2 holonomy, including the moduli stabilization, calculation of the moduli masses and decays and the entropy generated, etc (see [23]). Importantly, in order to obtain about the right relic density, the LSP must be a wino or wino-like particle, with a large annihilation cross section of about $3 \times 10^{-24} \text{ cm}^3/\text{sec}$. Such an LSP naturally arises in the G_2 -MSSM [24], where the tree level gaugino masses are suppressed by the supersymmetry breaking mechanism to be of order the gaugino masses from the anomaly mediation contribution. Thus the non-thermal history and a wino LSP go together and give a consistent picture for dark matter from the compactified string theory. Also encouraging is the fact that the PAMELA satellite data on positrons and antiprotons (which was reported after [24]) can be consistently described by a wino LSP [10].

One might wonder if the moduli “reheat temperature”, effectively the temperature generated by the moduli decay, could be above the thermal freezeout temperature so that the thermal history could finally take over. Unfortunately, the associated temperature is too small for a thermal history to develop. To see this let $T_{RH} \sim 10 \text{ MeV}$ for $m_{3/2} \sim 30 \text{ TeV}$, which follows from $T_{RH} \sim \sqrt{\Gamma_\phi m_{pl}} \sim \frac{m_{3/2}^{3/2}}{m_{pl}^{1/2}}$. Then if $m_{3/2}$ were larger by even an order of magnitude, T_{RH} would grow by a factor of order $(10)^{3/2} \sim 30$, so it would still be small compared to the usual thermal freezeout temperature $T_{fr} \sim \text{few GeV}$.

Thus very generally string theories with stabilized moduli having multi-TeV scale masses (or lighter) will have a non-thermal cosmological history, and a relic density of wino-like dark matter generated by moduli decay rather than thermal freezeout is the preferred solution with LSP dark matter. A fine-tuned period of late inflation may allow a way to evade this generic conclusion for some theories, but an inflaton with just the right properties must be found in such a theory.

UPPER LIMIT ON $m_{3/2}$

In theories where the relic density is indeed generated from moduli decay, it turns out there is an upper limit on $m_{3/2}$, because the universe would be overclosed if $m_{3/2}$ were too large. Since $\Gamma_\phi \sim m_{3/2}^3/m_{pl}^2$, $H(T_{RH}) \sim \Gamma_\phi$ and $T_{RH} \sim \sqrt{\Gamma_\phi m_{pl}}$, ρ/s for the dark matter is

$$\rho/s \sim \frac{M_{LSP}}{\langle \sigma v \rangle m_{3/2}^{3/2} m_{pl}^{1/2}} \quad (10)$$

M_{LSP} is the lightest eigenvalue of the neutralino mass matrix, which also contains μ and the Higgs vev, two mass scales not explicitly scaling as $m_{3/2}$. In a full theory we expect both

of these to vanish when $m_{3/2}$ vanishes. For example, radiative EWSB is not possible without supersymmetry breaking. Once $m_{3/2}$ is large compared to M_Z , the off-diagonal term in the neutralino mass matrix can be neglected. If $\mu \sim m_{3/2}$ the LSP is mostly gaugino and proportional to $m_{3/2}$, while if μ is small, there is Higgsino mixing in the LSP, but the LSP mass is again essentially proportional to $m_{3/2}$. More generally, in supergravity theories all masses will be proportional to $m_{3/2}$. Since $\langle \sigma v \rangle \sim M_{LSP}^{-2}$, it will decrease as $m_{3/2}^2$. Therefore

$$\rho/s \propto m_{3/2}^{3/2}/m_{pl}^{1/2} \quad (11)$$

If the correct relic density is obtained for, say $m_{3/2} \sim 50 \text{ TeV}$, a value of order 5 times larger for $m_{3/2}$ will overclose the universe, so $m_{3/2} \lesssim 250 \text{ TeV}$ is required. Any theory where all masses are proportional to $m_{3/2}$ will give a similar result.

GAUGE MEDIATION SUPERSYMMETRY BREAKING

Our results suggest that any approach to supersymmetry breaking that originates in a string theory with moduli that has a gravitino mass less than about 30 TeV will have the problems described above, the moduli and gravitino problems. Thus, one would conclude that gauge mediated supersymmetry breaking, which typically has a much lighter gravitino and therefore light moduli, does not generically arise if our universe is described by a compactified string theory with stabilized moduli.

HEAVY SCALARS, LIGHT GAUGINOS, LHC, AND RARE DECAYS

All superpartner masses in gravity mediated supersymmetry breaking are proportional to $m_{3/2}$. Scalar masses generically will have values about equal to $m_{3/2}$, but gaugino masses are often suppressed, usually because the main source of supersymmetry breaking does not couple at tree level with the gaugino masses, and they are zero in the supersymmetry limit. Several phenomenological consequences follow from these properties of generic string theory vacua.

At LHC the scalar superpartners should not be observed directly. The gauginos, in particular the gluino, the lightest two neutralinos (including the LSP), and the lighter chargino will be observed. The gaugino spectrum is typically compressed, e.g. in pure anomaly mediation with light scalars the ratio of gluinos to winos is 9, while in the G_2 case with heavy scalars it is about half that. Because the squark masses at the weak scale are given by running from the gravitino mass scale there are effects on gluino branching ratios even though the scalars cannot be directly observed, with a large BR of a gluino into top quarks (so gluino pairs often have 4 tops per event), and in general considerably larger BRs to channels with final b

quarks, leading to rich LHC physics [25]. Any effect on decays or moments of quarks and leptons that can only occur from loops should not differ from its Standard Model value significantly; in particular $g_\mu = 2$, $B_s \rightarrow \mu^+ \mu^-$, a charge asymmetry from B_s -mixing like-sign dimuons, and other effects should all take on their SM values. Models can of course be constructed with scalars ~ 1 TeV, and we are not aware of any study of how non-generic or unlikely such models are to arise in string theory.

Some predictions depend on how the μ problem is solved in string theory, which is rather poorly understood. On the one hand the μ term in the superpotential must vanish so μ does not have a string scale value, presumably because of a symmetry. But μ and the supersymmetry breaking $B\mu$ terms must be non-zero so the symmetry that protects μ must be broken. When that symmetry is broken doublet-triplet splitting must be preserved, the proton must not decay too rapidly, and the LSP must have a lifetime longer than about 10^{26} sec [26]. If the resulting value of μ is similar to the value of M_2 that gives the wino mass, then there is a light higgsino that mixes into the LSP and the light chargino. A higgsino admixture in the neutralino mass matrix gives an off diagonal term that dilutes the wino, and necessarily mixes in some bino as well. The main observable affected by the small μ is actually the scattering cross section in direct detection experiments. For a pure wino LSP the cross-section for LSP-proton scattering is below 10^{-46} cm², while with a higgsino admixture it can get as large as a *few* $\times 10^{-44}$ cm². Thus the Xenon100 measurement [27] will determine the allowed higgsino mixture in the wino, and approximately measure the value of μ .

CONCLUSIONS

We have argued that if our universe is described by a compactified string theory then the presence of stabilized moduli would likely imply that the cosmological history is non-thermal before BBN. In particular, dark matter can be produced from moduli decays and generically has to be wino-like in order to have a consistent abundance. The analysis that leads to these results sharpens the arguments that there is always a modulus with mass of order the gravitino mass or less in such theories. These plus cosmological considerations emphasise some difficulties in realizing gauge mediated supersymmetry breaking in string theory. We also described an upper limit on the gravitino mass of order several hundred TeV. The appendix contains a study of the known examples of string theory vacua with stabilized moduli and shows they agree with the results above.

ACKNOWLEDGEMENTS

We appreciate helpful conversations with Shanta de Alwis, Konstantin Bobkov, Frederik Denef, Michael Douglas, Daniel Feldman, Piyush Kumar, Aaron Pierce, and Fernando

Quevedo. We would also like to thank C. Scrucca and J. Louis for pointing out an error in the previous version of this article. B.A. is grateful to the University of Michigan Physics Department and MCTP for support, and E.K. is grateful to the String Vacuum Project for travel support and for a String Vacuum Project Graduate Fellowship funded through NSF grant PHY/0917807. This work was supported by the DOE Grant #DE-FG02-95ER40899.

APPENDIX A

For pedagogy we begin with a few toy (non-string theory) examples to demonstrate the validity of the above results.

Consider the Polonyi model, which has one field ϕ with G given by

$$G = m_{pl}^2 \phi \bar{\phi} + m_{pl}^2 \log \left| \frac{\mu^2}{m_{pl}^2} (\phi - \beta) \right|^2 \quad (12)$$

where $\beta = \sqrt{3} - 2$. The vacuum expectation value of ϕ is given by

$$\langle \phi \rangle = \langle \bar{\phi} \rangle = \sqrt{3} - 1 \quad (13)$$

and u and r are

$$u = 2\sqrt{3}m_{pl}^2, \quad r = 0 \quad (14)$$

The two eigenvalues of the mass matrix are

$$\begin{aligned} m_1^2 &= m_{3/2}^2 (2\sqrt{3}) = m_{3/2}^2 \frac{u}{m_{pl}^2} \\ m_2^2 &= m_{3/2}^2 (4 - 2\sqrt{3}) = m_{3/2}^2 (4 - \frac{u}{m_{pl}^2}) \end{aligned} \quad (15)$$

in agreement with our general result Eq. (7). The bounds are reached since the eigenvectors are given by the sGoldstino directions.

This provides as an illustration of how one might avoid our result by going to a non-string theory without moduli stabilization, by creating a model with large r and have scalar masses much heavier than the gravitino mass. Following [28], we add a higher dimensional operator to the above Kahler potential

$$G = m_{pl}^2 \phi \bar{\phi} + m_{pl}^4 \frac{(\phi \bar{\phi})^2}{\Lambda^2} + m_{pl}^2 \log \left| \frac{\mu^2}{m_{pl}^2} (\phi - \frac{1}{\sqrt{3}}) \right|^2. \quad (16)$$

with the new scale $\Lambda \ll m_{pl}$. There is a minimum of this potential where ϕ has a small vacuum expectation value. To leading order in $\frac{\Lambda}{m_{pl}}$

$$\langle \phi \rangle = \langle \bar{\phi} \rangle \approx \frac{\Lambda^2}{2\sqrt{3}m_{pl}^2}. \quad (17)$$

In this vacuum $r = -12 \frac{m_{pl}^4}{\Lambda^2}$, and one find after diagonalizing the scalar mass matrix that

$$m_1^2 = m_2^2 = 4\mu^2 \frac{m_{pl}^2}{\Lambda^2} = 12 \frac{m_{pl}^2}{\Lambda^2} m_{3/2}^2 = -\frac{r}{m_{pl}^2} m_{3/2}^2 \gg m_{3/2}^2. \quad (18)$$

Thus the scalar masses are at the bounds given by Eq. (6) and are much heavier than the gravitino mass. The field here has explicit couplings that are not moduli-like and if one tries to embed this model in string theory new problems arise – see Ex. 6 [29].

APPENDIX B

We now wish to examine examples we know of of string theory models in which all moduli are stabilized to gain further insight.

Ex. 1 Simple KKL Model

This example [30] has all complex structure moduli stabilized by fluxes and a single Kahler modulus and axion stabilized by non-perturbative corrections. The vacuum energy is tuned by adding what amounts to a D-term potential. We didn't consider D-terms above, but we can incorporate them into the discussion based on these examples. Both the Kahler modulus and the axion obtain masses of order $20 \times m_{3/2}$ in this simple model. Here $r \sim 0$, but the kinetic terms for moduli are suppressed by approximately $(\frac{1}{10})^2$, lifting the moduli masses above $\sqrt{2}m_{3/2}$. This scenario allows for a lighter gravitino $m_{3/2} \sim 1$ TeV, but the moduli will still dominate the cosmic energy density for times up to BBN and beyond.

The non-thermal cosmology of mirage-mediated supersymmetry breaking in the KKL context was discussed [32]. Late decay of moduli produce an abundance of Bino-like LSPs, which annihilate rapidly through the pseudo-scalar Higgs resonance.

Ex. 2 LARGE Volume IIB models

These examples [14] have complex structure moduli stabilized by fluxes and Kahler moduli stabilized by perturbative corrections. The vacuum energy is, naively, negative, though there might be mechanisms which generate the necessary positive contributions. The basic LARGE volume model has two Kahler moduli τ_b and τ_s . In the vacuum, $m_{3/2} \sim \frac{m_{pl}}{V}$, where V is the volume of the extra dimensions (divided by l_s^6). The masses of the moduli are given by $m_{\tau_b} \sim \frac{m_{3/2}}{V^{1/2}}$ and $m_{\tau_s} \sim m_{3/2} \log m_{pl}/m_{3/2}$. τ_b is much lighter than the gravitino and τ_s is an order of magnitude larger. Note that the suppression of the τ_b mass in this case can be shown to be from a direct cancellation in Eq. (6) as $r/m_{pl}^2 = 2 - O(1/V)$. In these models, unless $V \leq 10^9$, τ_b generically suffers from the cosmological moduli problem. In all cases, the early Universe is dominated by moduli oscillations. More recently, it has been realised that, by adding a third Kahler modulus, the observable sector supersymmetry breaking masses are suppressed relative to the gravitino mass, requiring $10^8 \text{ GeV} \leq m_{3/2} \leq 10^{11} \text{ GeV}$ [31]. Again, in all such cases, τ_b dominates the pre-BBN cosmic energy density.

Ex. 3 M theory and Type IIA flux Vacua

These examples [33] use fluxes to stabilize all the moduli. All these vacua have a negative cosmological constant and it seems difficult to add additional sources which could change

that. The moduli masses are all of order the gravitino mass.

Ex. 4 M theory on Manifolds of G_2 holonomy without flux.

These examples [23] are based upon the idea, which goes back to Witten and others [34], that strong dynamics in the hidden sector generates a potential which breaks supersymmetry and generates a hierarchically small scale (related to the weak scale). In the M theory context it has been shown that, additionally, the potential generated by such hidden sector dynamics can stabilize *all* the moduli fields. The minimum of the potential has positive energy. The moduli spectrum for these examples has been studied in detail in [23]. All moduli but one have masses of order $m_{3/2}$, the remaining one having a somewhat larger mass. Hence, again, the moduli dominate the early cosmological history but decay before BBN.

Ex. 5 Type IIB flux vacua with non-perturbative effects

These examples [35] apply the ideas of [24] to stabilize all Kahler moduli and obtain a vacuum with positive vacuum energy self consistently. These examples all have moduli whose masses are of order $m_{3/2}$ and hence will dominate the early Universe.

Ex. 6 Gauge Mediation in String theory?

In gauge mediation, the gravitino mass is relatively low and can be as small as an eV. Generically one expects that there are moduli whose masses are comparable to $m_{3/2}$. Since their lifetimes are so long, these moduli will dominate the Universe for many years and will not be able to reheat it to a temperature high enough for BBN to start (see [36] for a discussion on BBN constraints). Usually, when one considers gauge mediation, one implicitly assumes that moduli can be decoupled from the gravitino mass scale and then ignored, but our results indicate that such assumptions are perhaps too strong. Attempts at realizing a supergravity model derived from string theory with both moduli stabilization and gauge mediation are described in [29], based on earlier works of [28, 37]. These models essentially couple a Type IIB Kahler moduli sector to a gauge mediation model which is assumed to arise from a configuration of branes on the Calabi-Yau of the sort described in [38]. The authors of [29] explain that it is quite difficult to find a model in which gauge mediation effects are not overcome by those of gravity mediation, when the cosmological constant is tuned to zero. In any case, if one examines the moduli masses in those examples one finds that the moduli whose masses are dominated by D -terms have masses much larger than $m_{3/2}$, but those whose masses are dominated by F -terms have masses of order $m_{3/2}$. Therefore, generally one has moduli which lead to a non-thermal cosmological history.

APPENDIX C

Theorem Let H be an $n \times n$ positive definite (Hermitian) matrix and λ_1 be the smallest eigenvalue of H . Then

$$\lambda_1 \leq \xi^\dagger H \xi$$

for any unit vector ξ .

Proof Let U be the unitary matrix that diagonalizes H , i.e.,

$$U^\dagger H U = H_D = \begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \lambda_n \end{pmatrix}$$

where the eigenvalues, λ_i , are ordered $\lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_n$. Let ξ be any unit vector and define the vector

$$\chi = U^\dagger \xi$$

which is also has unit length since the unitary transformation, U , is an isometry. It then follows that

$$\lambda_\xi \equiv \xi^\dagger H \xi = \chi^\dagger U^\dagger H U \chi = \chi^\dagger H_D \chi = \sum_{i=1}^n \lambda_i |\chi_i|^2.$$

We wish show that $\lambda_1 \leq \lambda_\xi$. Assume this is false, i.e., $\lambda_\xi < \lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_n$. Then,

$$\lambda_\xi = \sum_{i=1}^n \lambda_i |\chi_i|^2 > \sum_{i=1}^n \lambda_\xi |\chi_i|^2 = \lambda_\xi \sum_{i=1}^n |\chi_i|^2 = \lambda_\xi$$

which is a contradiction. Therefore,

$$\lambda_1 \leq \lambda_\xi.$$

-
- [1] G. D. Coughlan, W. Fischler, E. W. Kolb, S. Raby, G. G. Ross, Phys. Lett. **B131**, 59 (1983). B. de Carlos, J. A. Casas, F. Quevedo, E. Roulet, Phys. Lett. **B318**, 447-456 (1993). [hep-ph/9308325]. M. Dine, L. Randall, S. D. Thomas, Phys. Rev. Lett. **75**, 398-401 (1995). [hep-ph/9503303].
- [2] T. Moroi, L. Randall, Nucl. Phys. **B570**, 455-472 (2000). [hep-ph/9906527].
- [3] B. S. Acharya, P. Kumar, K. Bobkov, G. Kane, J. Shao, S. Watson, JHEP **0806**, 064 (2008). [arXiv:0804.0863 [hep-ph]].
- [4] S. Weinberg, Phys. Rev. Lett. **48**, 1776-1779 (1982). J. R. Ellis, D. V. Nanopoulos, M. Quiros, Phys. Lett. **B174**, 176 (1986). M. Kawasaki and T. Moroi, Prog. Theor. Phys. **93**, 879 (1995) [arXiv:hep-ph/9403364].
- [5] G. D. Coughlan, R. Holman, P. Ramond, G. G. Ross, Phys. Lett. **B140**, 44 (1984). L. Randall and S. D. Thomas, Nucl. Phys. B **449**, 229 (1995) [arXiv:hep-ph/9407248]. D. H. Lyth, E. D. Stewart, Phys. Rev. **D53**, 1784-1798 (1996). [hep-ph/9510204].
- [6] T. Moroi, H. Murayama, M. Yamaguchi, Phys. Lett. **B303**, 289-294 (1993). A. de Gouvea, T. Moroi and H. Murayama, Phys. Rev. D **56**, 1281 (1997) [arXiv:hep-ph/9701244].
- [7] J. P. Conlon, F. Quevedo, JCAP **0708**, 019 (2007). [arXiv:0705.3460 [hep-ph]].
- [8] B. S. Acharya, G. Kane, S. Watson, P. Kumar, Phys. Rev. **D80** (2009) 083529. [arXiv:0908.2430 [astro-ph.CO]].
- [9] O. Adriani *et al.* [PAMELA Collaboration], Nature **458**, 607 (2009) [arXiv:0810.4995 [astro-ph]].
- [10] P. Grajek, G. Kane, D. J. Phalen, A. Pierce, S. Watson, [arXiv:0807.1508 [hep-ph]]. J. Hisano, M. Kawasaki, K. Kohri and K. Nakayama, Phys. Rev. D **79**, 063514 (2009) [Erratum-ibid. D **80**, 029907 (2009)] [arXiv:0810.1892 [hep-ph]]. G. Kane, R. Lu, S. Watson, Phys. Lett. **B681**, 151-160 (2009). [arXiv:0906.4765 [astro-ph.HE]]. D. Feldman, Z. Liu, P. Nath and B. D. Nelson, Phys. Rev. D **80**, 075001 (2009) [arXiv:0907.5392 [hep-ph]].
- [11] P. Fox, A. Pierce, S. D. Thomas, [hep-th/0409059]. B. S. Acharya, K. Bobkov, P. Kumar, [arXiv:1004.5138 [hep-th]].
- [12] F. Denef and M. R. Douglas, JHEP **0503**, 061 (2005) [arXiv:hep-th/0411183]. M. Gomez-Reino and C. A. Scrucca, JHEP **0605**, 015 (2006) [arXiv:hep-th/0602246].
- [13] L. Covi, M. Gomez-Reino, C. Gross *et al.*, JHEP **0903**, 146 (2009). [arXiv:0812.3864 [hep-th]].
- [14] V. Balasubramanian, P. Berglund, J. P. Conlon, F. Quevedo JHEP **0503**, 007 (2005). [hep-th/0502058].
- [15] A. Strominger, Commun. Math. Phys. **133**, 163-180 (1990). C. L. Wang, Doc. Math. **8**, 577-590 (2003).
- [16] T. Trenner, P. M. H. Wilson, [arXiv:0902.4611].
- [17] B. S. Acharya, K. Bobkov, [arXiv:0810.3285 [hep-th]].
- [18] M. Dine, Y. Nir and Y. Shadmi, Phys. Lett. B **438**, 61 (1998) [arXiv:hep-th/9806124].
- [19] S. Watson, [arXiv:0912.3003 [hep-th]].
- [20] R. H. Brandenberger, C. Vafa, Nucl. Phys. **B316**, 391 (1989). T. Battefeld, S. Watson, Rev. Mod. Phys. **78**, 435-454 (2006). [hep-th/0510022].
- [21] E. Witten, Phys. Lett. **B149**, 351-356 (1984). J. E. Kim, Phys. Rept. **150**, 1-177 (1987). P. Svrcek and E. Witten, JHEP **0606** (2006) 051 [arXiv:hep-th/0605206]. A. Arvanitaki, S. Dimopoulos, S. Dubovsky, N. Kaloper, J. March-Russell, [arXiv:0905.4720 [hep-th]].
- [22] S. P. de Alwis, [arXiv:1006.0949 [hep-th]].
- [23] B. S. Acharya, K. Bobkov, G. L. Kane, J. Shao, P. Kumar, Phys. Rev. **D78**, 065038 (2008). [arXiv:0801.0478 [hep-ph]].
- [24] B. S. Acharya, K. Bobkov, G. Kane, P. Kumar, D. Vaman, Phys. Rev. Lett. **97** (2006) 191601. [hep-th/0606262]. B. S. Acharya, K. Bobkov, G. L. Kane, J. Shao, P. Kumar, Phys. Rev. **D76**, 126010 (2007). [hep-th/0701034].
- [25] B. S. Acharya, P. Grajek, G. L. Kane, E. Kuflik, K. Suruliz, L. T. Wang, [arXiv:0901.3367 [hep-ph]]. D. Feldman, G. Kane, R. Lu and B. D. Nelson, Phys. Lett. B **687**, 363 (2010) [arXiv:1002.2430 [hep-ph]]. G. F. Giudice, T. Han, K. Wang and L. T. Wang, arXiv:1004.4902 [hep-ph].
- [26] V. Berezhinsky, A. Masiero, J. W. F. Valle, Phys. Lett. **B266**, 382-388 (1991). E. A. Baltz, P. Gondolo, Phys. Rev. **D57**, 7601-7606 (1998). [hep-ph/9704411].
- [27] E. Aprile *et al.* [XENON100 Collaboration], arXiv:1005.0380 [astro-ph.CO].
- [28] R. Kitano, Phys. Lett. **B641**, 203-207 (2006). [hep-ph/0607090]. M. Ibe, R. Kitano, Phys. Rev. **D75**, 055003 (2007). [hep-ph/0611111].
- [29] E. Dudas, Y. Mambrini, S. Pokorski, A. Romagnoni, M. Trapletti, JHEP **0903**, 011 (2009). [arXiv:0809.5064 [hep-th]].
- [30] S. Kachru, R. Kallosh, A. D. Linde and S. P. Trivedi, Phys. Rev. D **68** (2003) 046005 [arXiv:hep-th/0301240].
- [31] R. Blumenhagen, J. P. Conlon, S. Krippendorff, S. Moster, F. Quevedo, JHEP **0909**, 007 (2009). [arXiv:0906.3297 [hep-th]].
- [32] M. Nagai and K. Nakayama, Phys. Rev. D **76**, 123501 (2007)

- [arXiv:0709.3918 [hep-ph]].
- [33] B. S. Acharya, [hep-th/0212294]. G. Villadoro and F. Zwirner, JHEP **0506**, 047 (2005) [arXiv:hep-th/0503169]. B. S. Acharya, F. Denef, R. Valandro, JHEP **0506**, 056 (2005). [hep-th/0502060]. O. DeWolfe, A. Giryavets, S. Kachru, W. Taylor, JHEP **0507**, 066 (2005). [hep-th/0505160]. B. S. Acharya, F. Benini, R. Valandro, JHEP **0702**, 018 (2007). [hep-th/0607223].
- [34] B. S. Acharya and E. Witten, arXiv:hep-th/0109152.
- [35] K. Bobkov, V. Braun, P. Kumar, S. Raby [arXiv:1003.1982 [hep-th]].
- [36] G. B. Gelmini and P. Gondolo, Phys. Rev. D **74**, 023510 (2006) [arXiv:hep-ph/0602230].
- [37] Z. Lalak, S. Pokorski, K. Turzynski, JHEP **0810**, 016 (2008). [arXiv:0808.0470 [hep-ph]].
- [38] D. E. Diaconescu, B. Florea, S. Kachru and P. Svrcek, JHEP **0602**, 020 (2006) [arXiv:hep-th/0512170].